Math Virtual Learning

Calculus AB

Integration of Special Functions

May 14, 2020

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Lesson: May 14, 2020

Objective/Learning Target:

Lesson 4 Integrals Review

Students will evaluate integrals of special functions such as ln(x), e^x, and inverse trig functions.

Warm-Up:

Note: This is a review. For more examples refer back to your notes.

Watch Videos: Integrals of exponentials

Integrals of logarithms

Integrals of inverse trig functions

Notes:

THEOREM 5.5 Log Rule for Integration

Let u be a differentiable function of x.

1. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

THEOREM 5.12 Integration Rules for Exponential Functions

Let u be a differentiable function of x.

1. $\displaystyle\int e^x\,dx = e^x + C$
2. $\displaystyle\int e^u\,du = e^u + C$

THEOREM 5.17 Integrals Involving Inverse Trigonometric Functions

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$
2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

Examples:

Find the integral $\int xe^{-x^2}dx$.

Solution.

Using the substitution $u = -x^2$, we have

$$du = d\left(-x^2\right) = -2xdx.$$

Note that

$$xdx = -\frac{du}{2},$$

so we can rewrite the integral in terms of the variable u and solve it:

$$\int xe^{-x^2}dx = \int e^u\left(-\frac{du}{2}\right) = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C = -\frac{e^{-x^2}}{2} + C.$$

Examples:

Evaluate the integral $\int \frac{\sin x}{1-\cos x} dx$.

Solution.

We make the substitution $u = 1 - \cos x$. Hence

$$du = -(-\sin x)\, dx = \sin x dx.$$

This gives

$$\int \frac{\sin x}{1 - \cos x} dx = \int \frac{du}{u} = \ln|u| + C = \ln|1 - \cos x| + C.$$

Examples:

Compute the integral $\int \frac{xdx}{1+x^4}$.

Solution.

We can try the substitution $u = x^2$. Then

$$du = 2xdx, \quad \Rightarrow xdx = \frac{du}{2}.$$

Hence, the integral is equal to

$$\int \frac{xdx}{1+x^4} = \int \frac{\frac{du}{2}}{1+u^2} = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\arctan u + C = \frac{1}{2}\arctan x^2 + C.$$

Practice:

1) Find the integral $\int \frac{x+1}{x^2+2x-5}\,dx$.

2) Compute the integral $\int e^{\frac{x}{2}}\,dx$.

Answer Key:

Once you have completed the problems, check your answers here.

1)

Solution.

We make the substitution $u = x^2 + 2x - 5$. Then $du = 2xdx + 2dx = 2\left(x + 1\right)dx$ or $\left(x + 1\right)dx = \frac{du}{2}$. The integral is easy to calculate with the new variable:

$$\int \frac{x+1}{x^2+2x-5}dx = \int \frac{\frac{du}{2}}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x-5| + C.$$

Answer Key:

Once you have completed the problems, check your answers here.

2) *Solution.*

Let $u = \frac{x}{2}$. Then

$$du = \frac{dx}{2}, \quad \Rightarrow dx = 2du.$$

So now we can easily integrate:

$$\int e^{\frac{x}{2}} dx = \int e^u \cdot 2du = 2\int e^u du = 2e^u + C = 2e^{\frac{x}{2}} + C.$$

Additional Practice:

[Interactive Practice]

[Extra Practice with Answers]

[More Extra Practice with Answers]